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Automatic Shape Control of Deformable Rods Based on Data-Driven Implicit Sensorimotor Models

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Abstract— **In this work, we propose a general approach to design shape servoing controller for manipulating the deformable object into the desired shape. The raw visual feedback data will be processed using regression method to identify the parameters of the continuous geometric model defined as the shape feature based on the specific task, which is able to globally represent the object. The derivation of analytical pose-shape Jacobian matrix based on implicit functions is provided for, sometimes, it is not easy to obtain the explicit mapping of object deformation and robotic pose. Then, the shape servoing controller based on velocity is designed using the derived pose-shape Jacobian matrix to enable the robot to manipulate the deformable object into the desired shape.**

Index Terms— **Robotics, visual servoing, deformable objects, shape control.**

I. Introduction

In the first decades of the rise of robotics, manipulating rigid body obtained widely concerned because its ideal model is able to simplify the system and help researchers focus on studying functions of robotic manipulators. With the researches about rigid body increasingly accumulated in robotics, there are more and more demands for application of robots manipulating soft object, such as domestic services robot, food engineering robot, medical surgery robot, etc. Following this trend, many soft objects manipulation tasks are proposed, such as shape estimation, physical parameters identification, motion planning, deformation control. However, over-classified tasks in terms of the shape of deformable objects (linear, planar, volumetric, etc.) and robotic manipulation behavior (knotting, bending, folding, cutting, etc.) result in plenty of repetitive work and hinder researchers' communication. To address this issue, this work aims at proposing a general approach to analyse and solve the deformable object manipulation control problem.

At the very beginning when manipulating deformable objects started to attract attention, most researchers preferred to model the deformable object based on accurate physical mechanism. However, it is impossible to exactly analyse force and deformation and estimate physical parameters for each object to be manipulated since the soft object may be non-homogeneous. To avoid prior knowledge, a new trend, namely model-free shape servoing control [1], was rising. Shape servoing control makes manipulator deform the soft object from the current state into the desired shape based on quantified shape feature to describe the deformation of the soft object, such as points, angles, curvature, contour, catenary, etc. Learning or estimating techniques to obtain the relationship between deformation and robotic pose are usually applied to design controllers. Although these methods don't require physical models, collecting data and training are necessary and repeated with the object changing in the each specific task, which highly increases workload. Hence, this work uses continuous geometry, such as curve or surface, to globally and analytically describe the deformable object, and builds up the mapping of deformation and robotic pose based on geometric relationship.

Mostly, soft object control methods utilize 2D images as the sensor feedback for the development of 2D image processing is early, numerous, and mature. However, with the development of the depth camera and the relevant processing methods, the application of 3D data with higher dimensional information starts to show its advantages. Since the unorganized raw feedback data from an RGB-D sensor is not able to tell the relationship of one point in the real world with the others, the classic point cloud processing approach is building topology geometric mesh to simulate and reconstruct their physical relationship. Although it can obtain a relatively precise surface structure, building precise topology mapping for point cloud results in heavy computing load and redundant information. Therefore, this paper proposes a method able to deal with high dimensional visual feedback using unorganized raw visual data to optimize the data processing. A simple continuous geometric model is selected as the shape feature, whose parameters will be identified through the conventional regression methods based on raw sensing data. Continuous geometric model has lower variables of shape feature than gain topological mesh, which improve the controllability of the elastic deformation.

To sum up, the main contributions of the proposed method are: 1) designing model-free controller to avoid the demand for the prior knowledge, 2) using unorganized raw visual data to promote the computing speed, 3) selecting simple geometric curve as the shape feature to express the global deformation of the linear soft object, and 4) proposing a general methodology with implicit function theorem to design shape servoing model-free controller.

II. Methodology

Shape servoing control is pretty complicated since the deformable object can be linear, surficial, volumetric, and there are lots of sensors classified by the feedback data, such as RGB image, point cloud, etc. Meanwhile, even for the same setup, we expect the robotic manipulator to achieve different behaviours, like poking or grasping a foam, folding a cloth,

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banding a soft stick, etc. Different combinations possibly have different optimal solutions based on its properties and limitations. But there is a huge workload to design method for every task. In this section, we propose a methodology to solve these problems in a general and global way.

A. Overview

First, determine the research target, that is, the type of soft object, the feedback data from sensor, the desired result, etc. Then, according to the this target, select a suitable geometric model to approximate the shape of the object. For example, when we study a linear deformable object, we could use a conic section on a spatial plane if the soft object is banded and use a helix if the soft object is twisted.

Second, analyse the geometric relationship between the soft object and the pose of robotic manipulator relying on the selected shape feature. Then, compute the analytical the pose-shape Jacobian matrix by taking the partial derivatives of the pose of the manipulator with respect to the shape feature parameters. Since implicit functions are the most likely used to describe the geometric relationship, implicit function theorem is introduced to get the pose-shape Jacobian matrix.

Finally, design a velocity-based controller in the task space using the obtained analytical pose-shape Jacobian matrix, for Jacobian matrix is a tool to describe the relationship of velocity.

After the analysis is done, the control law is able to be used to update the control command for the manipulator in the each loop. Once the control process starts, the raw data should be online fitted to the selected geometric model to identify shape feature parameters. This is a typical regression problem, which could be either linear or nonlinear. The frequent solutions are Least Square Method (LSM), Gradient Descent, Newton's method, Quasi-Newton Methods, Conjugate Gradient, etc.

B. Online identification of shape feature

As we mentioned, the regression method will be utlized to fit raw data to geometric model identifying shape feature in every control loop. In this paper, we choose LSM for it is simple and fast. LSM is a classical linear regression algorithm whose core is to identify the parameters of the model to minimize the sum of squared residual defined as the difference between the observed value by sample and the predicted value by model.

Let $\boldsymbol{x} = [x_1, \cdots, x_q]^T \in \mathbb{R}^q$ denote the pose of the robot end-effector, which is the feedback from the robotic manipulator. Denote m-dimensional parameters vector of shape feature as $\boldsymbol{y} = [y_1, ..., y_m]^T \in \mathbb{R}^m$ which is online identified during the control process. Let $\boldsymbol{S} = [\boldsymbol{s}_1, \cdots, \boldsymbol{s}_N] \in \mathbb{R}^{n\times N}$ denote the unorganized raw feedback from sensors (pixels in 2D RGB-image, point cloud on the soft object surface, etc.) where $\boldsymbol{s}_i$ is a single n-dimensional element of data. Assume there are N elements in set $\boldsymbol{S}$ used for parameter identification. The mapping between geometric information $\boldsymbol{s}$ and the shape feature parameter $\boldsymbol{y}$ satisfy

$$\boldsymbol{f}\left([\boldsymbol{s}^T, \boldsymbol{y}^T]^T\right) = \boldsymbol{0}_{l\times 1}, \tag{1}$$

where $\boldsymbol{y}$ is the shape feature to be identified, $\boldsymbol{f}$ is an implicit equation set comprising l equations $\boldsymbol{f} = \{f_1, \cdots, f_l\}$, and $f_i (i = 1 \cdots l)$ is a twice continuously differentiable function. The fitting process is an unconstrained optimization problem

$$\min_{\boldsymbol{y}} \left\{ \sum_{i=1}^{l} f_i^2([\boldsymbol{s}^T, \boldsymbol{y}^T]^T) : \boldsymbol{s} \in \mathbb{R}^n, \boldsymbol{y} \in \mathbb{R}^m \right\}. \tag{2}$$

C. Derivation of analytical Jacobian matrix

Jacobian matrix is a tool to describe the velocity relationship which is obtained from the first-order partial derivatives of the displacement mapping. Traditionally, the mapping should be explicit, while it is hard to obtain for the selected geometric model. Hence, to derive a Jacobian matrix based on implicit mapping, we introduce implicit function theorem [2] as follows:

Lemma 1 (Implicit function theorem)**.** *Let $\boldsymbol{h} : \mathbb{R}^{q+m} \to \mathbb{R}^q$ be a continuously differentiable function of two sets of variables, $\boldsymbol{x} \in \mathbb{R}^q$ and $\boldsymbol{y} \in \mathbb{R}^m$: $\boldsymbol{h}\left([\boldsymbol{x}^T, \boldsymbol{y}^T]^T\right) = \boldsymbol{0}$. If the Jacobian matrix*

$$\boldsymbol{J}_{\boldsymbol{h},\boldsymbol{x}} = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}} \in \mathbb{R}^{q\times q} \tag{3}$$

is invertible, the Jacobian matrix of $\boldsymbol{x}$ with respect to $\boldsymbol{y}$ is given by the matrix product

$$\boldsymbol{J}_{\boldsymbol{x},\boldsymbol{y}} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{y}} = -\left[\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}\right]^{-1} \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{y}} \in \mathbb{R}^{q\times m} \tag{4}$$

For the general p dimensional mapping $\boldsymbol{h} : \mathbb{R}^{q+m} \to \mathbb{R}^p$, we compute the Jacobian matrix $\boldsymbol{J}_S$ using pseudoinverse trick.

D. Designing controller

Given the desired shape $\boldsymbol{y}_d$ and its deformation velocity $\dot{\boldsymbol{y}}_d$, a task space controller can be designed based on analytical Jacobian matrix $\boldsymbol{J}_S$ and online updating shape-feature parameters $\boldsymbol{y}$. The desired velocity of the manipulator end-effector can be obtained by

$$\dot{\boldsymbol{x}} = \boldsymbol{J}_S\left(\dot{\boldsymbol{y}}_d - \boldsymbol{K}(\boldsymbol{y} - \boldsymbol{y}_d)\right) \tag{5}$$

where $\boldsymbol{K}$ is a positive definite gain matrix. The desired end-effector trajectory $\boldsymbol{x}$ is obtained by numerical integration.

To proof the stability, we select a Lyapunov function $V = \frac{1}{2}\boldsymbol{e}^T\boldsymbol{e}$, where $\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{y}_d$. Take the derivative of V to yield $\dot{V} = \boldsymbol{e}^T\dot{\boldsymbol{e}}$, where $\dot{\boldsymbol{e}} = \dot{\boldsymbol{y}} - \dot{\boldsymbol{y}}_d$. Submit Eq.5 in it, we yield

$$\dot{\boldsymbol{e}} = \boldsymbol{J}_S^{\dagger}\dot{\boldsymbol{x}} - [\boldsymbol{J}_S^{\dagger}\dot{\boldsymbol{x}} + \boldsymbol{K}(\boldsymbol{y} - \boldsymbol{y}_d)] \quad = -\boldsymbol{K}\boldsymbol{e} \tag{6}$$

Submit Eq.6 into $\dot{V}$, the derivative is $\dot{V} = -\boldsymbol{e}^T\boldsymbol{K}\boldsymbol{e} \leq 0$. Then, the global stability of the controller is proofed.

References

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