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Trajectory tracking strategy of quadrotor with output delay

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Abstract:
A nonlinear output-feedback control strategy is devised using delayed-output observer for an underactuated quadrotor to track a given reference trajectory. The presented strategy consists of a delayed-output observer. A two-step observation algorithm is used to reconstruct the system conditions to compose the presented observer. By utilizing the presented strategy, the problem of time-delay is solved. The dynamic model of the quadrotor is presented firstly. To guarantee that the observation of the specific delay factors in the system is exponential convergent to zero under the given conditions. The advantages of the presented strategy are its practicability and extensive applications. Numerical results are presented for a practical tracking case under arbitrarily large initial and desired conditions.

Key Words: Trajectory tracing, Output delay, dynamic model

1 Introduction

The presented paper pays attention to the problem of output tracking for quadrotor with the delayed outputs. Under normal conditions, information between position and attitude is measured by GPS and inertial measurement unit (IMU). However there exists time delays between position information from satellites to aircraft and measurements of attitude by IMU and GPS. The former can be ignored because it's value is small. The main factor is calculation time by GPS and IMU. The primary difficulty with controlling quadrotor is that it's underactuated and transmitting delayed. Over the last decade, people attempted to put alternate methodologies into the use of the design of the quadrotor attitude and trajectory control. With the mathematical model of which is complex and it's difficult to design accurate controller to command it, and quadrotor has been used in many tasks as search and rescue, building exploration, security and inspection.Hence, researches between attitude and trajectory control are significant both in theory and practice. An integral predictive and nonlinear robust control strategy to solve trajectory tracking for a quadrotor has been presented in Guilherme V.Raffo[1].Abdelhamid Tayebi[3] has developed a new feedback control based on the compensation of the Coriolis, gyroscopic torques and the use of PD^2 feedback algorithm for exponential attitude stabilization. Erding Altug[4] presented a visual feedback strategy which used a ground camera as the primary sensor to estimate the pose(position and orientation) of the quadrotor. A novel nonlinear controller was presented in Travis Dierks[5] with using neural networks (NNs) and output feedback in the presence of this case. The problem studied in Fengjie Gao[6] was to design an algorithm which utilized the linear extended state observer to estimate uncertainties in quadrotor system. Besides, Lyapunov function was used to design the backstepping method to compensate for quadrotor uncertainties while satisfying output tracking error. Hieu Minh Trinh[7] discussed a comprehensive method based on the design of functional observers for linear systems having a time-varying in the state variables. The presented method was characterized by the features of being low-order and delay free. V Sundarapandian[8] investigated a math theory based on reduced order observer for nonlinear systems. The method studied in this paper possessed attractive features of being suited to Lyapunov stable nonlinear systems with a linear output equation, where the error convergence was established for the reduced order estimator for nonlinear systems using center manifold theory for flows. En-hui Zheng[9] presented a control method which was applicable for small quadrotor, based upon second order sliding mode control (2-SMC). To perform the pose and attitude tracking control of the quadrotor perfectly, the dynamical model of quadrotor was divided into two subsystems, a fully actuated subsystem and an underactuated subsystem.

2 Control Strategy

2.1 Dynamic modeling of a quadrotor

Generally, the dynamic model of the quadrotor is described by dual coordinate (Inertial coordinate frame and body coordinate frame) which is shown in Fig 1. Inertial coordinate frame is used to describe translation motion, while the body coordinate frame describes rotate motion.

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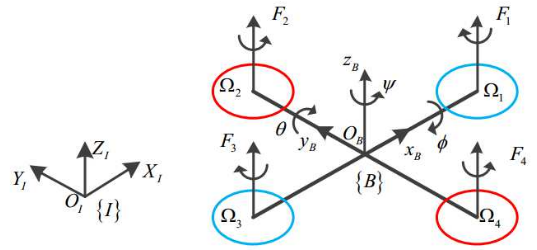

Fig.1. Inertial coordinate frame and body coordinate frame.

In Fig.1, the characteristic of the quadrotor is presented by the central symmetry and degustation figure. Four brushless motors and propellers constitute the power system. The sequence number of motors have been given in Fig.1. Among these four motors, motor 1 and 3 operate in the clockwise rotation, while motor 2 and 4 operate in the negative rotation. Propellers rotate as the motors to generate the respective lift force $F_i\left(i=1,2,3,4\right)$ which the quadrotor needs. When the rotate speeds of motors are equal, the reaction torques generated by four motors are balanced, the quadrotor keep balance. In contrast, the generated unbalanced torques will make the quadrotor rotate. By changing rotate speeds to change the individual lift force generated by motors to achieve the goal to change flight attitude and trajectory eventually. The paper is based on the crossing flight mode to build the dynamic model, describing attitude by Euler angles and modeled by Newton-Euler formula. Define $s=\left[x,y,z,\phi,\theta,\varphi\right]^T\in R^6$ as generalized coordinates. Where $\left[x,y,z\right]^T$ describes the position condition of aircraft, while $\left[\phi,\theta,\varphi\right]^T$ describes the attitude of the aircraft. ϕ is roll angle, θ is pitch angle and φ is yaw angle.

The rotation matrix **R** of translational motion from body frame **B** to inertial frame **I** shown by matrix(1).

$$\boldsymbol{R}=\boldsymbol{R}_x\left(\phi\right)\boldsymbol{R}_y\left(\theta\right)\boldsymbol{R}_z\left(\varphi\right)=\begin{bmatrix} c\varphi c\theta & c\varphi s\theta s\phi - c\phi s\varphi & c\varphi s\theta c\phi + s\varphi s\phi \\ c\theta s\varphi & s\varphi s\theta s\phi + c\phi c\varphi & c\phi s\varphi s\theta - c\varphi s\phi \\ -s\theta & c\theta s\phi & c\phi c\theta \end{bmatrix} \tag{1}$$

The transition matrix **T** of rotational motion from body frame **B** to inertial frame **I** shown by matrix(2).

$$\boldsymbol{T}=\begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \tag{2}$$

Based on Newton's second law, the kinetic equation of translational motion is the following.

$$m\dot{\boldsymbol{v}}=-mg\boldsymbol{e}_z+u_1\boldsymbol{R}\boldsymbol{e}_z$$

Where m is the quality of quadrotor, $\boldsymbol{v}=\left[v_x\ v_y\ v_z\right]^T$ $v_x\ v_y\ v_z$ is respective translational motion velocity along with axis in the inertial frame. g is gravitational acceleration, $\boldsymbol{e}_z=\left[0,0,1\right]^T$ and $u_1=\sum_{i=1}^{4}F_i=K_t\sum_{i=1}^{4}\Omega_i^2$ is the total lift force generated by four motors. K_t is the comprehensive lift force coefficient of each propeller. Ω_i $(i=1,2,3,4)$ is the rotate speed of each motor. And the kinetic equation of rotation motion is presented as.

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}=-\boldsymbol{\omega}\times\boldsymbol{J}\boldsymbol{\omega}+\boldsymbol{\tau}$$

Where $\boldsymbol{J}=\begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix}$ is inertial matrix. J_x is the rotational inertia with X axis. J_y is the rotational inertia of Y axis. J_z is the rotational inertia of Z axis. $-\boldsymbol{\omega}\times\boldsymbol{J}\boldsymbol{\omega}$ is the gyroscopic effect torque. $\boldsymbol{\omega}=\left[\omega_x\ \omega_y\ \omega_z\right]^T$, $\omega_x,\omega_y,\omega_z$ is respectively angle velocity along with axis in the body frame.

$$\boldsymbol{\tau}=\begin{bmatrix}\tau_\phi \\ \tau_\theta \\ \tau_\varphi\end{bmatrix}=\begin{bmatrix} lK_t\left(\Omega_4^2-\Omega_2^2\right) \\ lK_t\left(\Omega_3^2-\Omega_1^2\right) \\ K_d\left(\Omega_4^2+\Omega_2^2-\Omega_1^2-\Omega_3^2\right)\end{bmatrix}$$

Where $\boldsymbol{\tau}$ is the total control torque. K_d is the comprehensive resistance coefficient, τ_ϕ is roll torque component, τ_θ is pitch torque component and τ_φ is yaw torque component. l represents the proximity from the central of quadrotor to the central of trochanter. Eq.(3) is the dynamic model about $\left[x,y,z\right]^T$ and $\left[\phi,\theta,\varphi\right]^T$.

$$\begin{cases} \ddot{x}=\dfrac{u_1}{m}\left(\cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi\right) \\ \ddot{y}=\dfrac{u_1}{m}\left(\cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi\right) \\ \ddot{z}=\dfrac{u_1}{m}\cos\phi\cos\theta-g \\ \ddot{\phi}=\dfrac{l}{J_x}u_2 \\ \ddot{\theta}=\dfrac{l}{J_y}u_3 \\ \ddot{\varphi}=\dfrac{1}{J_z}u_4 \end{cases} \tag{3}$$

Where, u2 is the roll force, u3 is the pitch force and u4 is the yaw force.

2.2 Delayed-Output observer design

The paper sets six variables ($U_1U_2U_3U_4U_5U_6$) which are shown in Eq. (4) as the virtual controllers.

$$\begin{aligned} U_1&=\left(\cos\phi\sin\theta\cos\varphi+\sin\phi\sin\varphi\right)\frac{u_1}{m} \\ U_2&=\left(\cos\phi\sin\theta\sin\varphi-\sin\phi\cos\varphi\right)\frac{u_1}{m} \\ U_3&=\cos\phi\cos\theta\frac{u_1}{m}-g \\ U_4&=\frac{l}{J_x}u_2 \\ U_5&=\frac{l}{J_y}u_3 \\ U_6&=\frac{1}{J_z}u_4 \end{aligned} \tag{4}$$

Eq. (5) can be given by optimizing Eq.(4)

$$\begin{cases} u_1 = m\sqrt{U_1^2+U_2^2+\left(U_3+g\right)^2} \\ \phi_d = arcsin\left(\dfrac{m}{u_1}\left(U_1\sin\varphi_d - U_2\cos\varphi_d\right)\right) \\ \theta_d = arctan\left(\dfrac{1}{u_1+g}\left(U_1\cos\varphi_d + U_2\sin\varphi_d\right)\right) \\ u_2 = \dfrac{J_x}{l}U_4 \\ u_3 = \dfrac{J_y}{l}U_5 \\ u_4 = J_z U_6 \end{cases} \tag{5}$$

Where, ϕ_d is the desired roll angle, θ_d is the desired pitch angle and φ_d is the desired yaw angle. The dynamic model of the quadrotor is rewritten by Eq. (6).

$$\begin{aligned} \ddot{x} &= U_1 \\ \ddot{y} &= U_2 \\ \ddot{z} &= U_3 \\ \ddot{\phi} &= U_4 \\ \ddot{\theta} &= U_5 \\ \ddot{\varphi} &= U_6 \end{aligned} \tag{6}$$

However, there exists time delays in the practical dynamic model of the quadrotor. Here, we assume time delays coefficients ($\Delta_i \quad i=1,2\ldots6$) constants with the reference paper in Xinhua Wang[10].
Denote:

$$x_1=x, x_2=\dot{x}, y_1=y, y_2=\dot{y}, z_1=z, z_2=\dot{z}$$

$$\phi_1=\phi, \phi_2=\dot{\phi}, \varphi_1=\varphi, \varphi_2=\dot{\varphi}, \theta_1=\theta, \theta_2=\dot{\theta}:$$

$$\boldsymbol{S_1}=\left[x_1 \ \ x_2\right]^T \ \boldsymbol{S_2}=\left[y_1 \ \ y_2\right]^T \ \boldsymbol{S_3}=\left[z_1 \ \ z_2\right]^T$$

$$\boldsymbol{S_4}=\left[\phi_1 \ \ \phi_2\right] \ \ \boldsymbol{S_5}=\left[\theta_1 \ \ \theta_2\right] \ \boldsymbol{S_6}=\left[\varphi_1\varphi_2\right]$$

$$\boldsymbol{A}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\ \boldsymbol{H}=\begin{bmatrix}0\\ 1\end{bmatrix},\ \boldsymbol{C}=\begin{bmatrix}1 & 0\end{bmatrix}$$

Where **A** is the system matrix , **H** is the control matrix and **C** is the output matrix.

$$\overline{y_i}(t)=\boldsymbol{CS_i}\left(t-\Delta_i\right) \quad \boldsymbol{i}=1,2,3,4,5,6 \quad \boldsymbol{S_i}(t)=\begin{bmatrix}S_{i1}(t)\\ S_{i2}(t)\end{bmatrix}$$

Dynamic model is given by Eq.(7)

$$\dot{\boldsymbol{S}}_{\boldsymbol{i}}(t)=\boldsymbol{AS_i}(\boldsymbol{t})+\boldsymbol{H}U_i(t) \quad i=1,\ldots,6 \tag{7}$$

$\boldsymbol{S_i(t)}$ is the state vector while $U_i(t)$ is the control vector.
The estimated value of $\boldsymbol{S_i(t)}$ as $\boldsymbol{\hat{S}_i(t)}$ and we design the sub-observers as Eq. (8) and Eq. (9)follows.

$$\dot{\hat{\boldsymbol{S}}}_{\boldsymbol{i}}(t)=\boldsymbol{A}\hat{\boldsymbol{S}}_{\boldsymbol{i}}(t)+\boldsymbol{H}U_i(t)+e^{A\Delta_i}k_i\left(\overline{y_i}(t)-\boldsymbol{C}\hat{S}_i\left(t-\Delta_i\right)\right) \tag{8}$$

$$\begin{aligned}\dot{\hat{\boldsymbol{S}}}_{\boldsymbol{i}}\left(t-\Delta_i\right)=&\boldsymbol{A}\hat{\boldsymbol{S}}_{\boldsymbol{i}}\left(t-\Delta_i\right)+\boldsymbol{H}U_i\left(t-\Delta_i\right)\\ &+\boldsymbol{k_i}\left(\overline{y_i}(t)-\boldsymbol{C}\hat{\boldsymbol{S}}_{\boldsymbol{i}}\left(t-\Delta_i\right)\right)\end{aligned} \tag{9}$$

And

$$\boldsymbol{k_i}=\begin{bmatrix}k_{i1}\\ k_{i2}\end{bmatrix}$$

Where $\boldsymbol{k_i}$ is selected to make $\boldsymbol{A}-\boldsymbol{k_i}C$ Hurwitz matrix and estimate error is presented as Eq. (10).

$$\boldsymbol{\delta_i}(t)=\left[\delta_{i1}(t) \ \ \delta_{i2}(t)\right]^T=\left[\hat{S}_{i1}(t)-S_{i1}(t) \quad \hat{S}_{i2}(t)-S_{i2}(t)\right]^T \tag{10}$$

Convergence analysis is the following.

$$\dot{\boldsymbol{S}}_{\boldsymbol{i}}\left(t-\Delta_i\right)=\boldsymbol{AS_i}\left(t-\Delta_i\right)+\boldsymbol{H}U_i\left(t-\Delta_i\right) \tag{11}$$

By Eq.(10) ,we can get Eq.(12).

$$\dot{\boldsymbol{\delta}}_{\boldsymbol{i}}\left(t-\Delta_i\right)=\dot{\hat{\boldsymbol{S}}}_{\boldsymbol{i}}\left(t-\Delta_i\right)-\dot{\boldsymbol{S}}_{\boldsymbol{i}}\left(t-\Delta_i\right)=\left(\boldsymbol{A}-\boldsymbol{k_i C}\right)\boldsymbol{\delta_i}\left(t-\Delta_i\right) \tag{12}$$

The solution of Eq. (12) is Eq. (13).

$$\boldsymbol{\delta_i}\left(t-\Delta_i\right)=\boldsymbol{\delta_i}\left(t_0-\Delta_i\right)e^{(\boldsymbol{A}-\boldsymbol{k_i C})(t-t_0)} \tag{13}$$

Where, t0 is the initial time and $\dot{\delta}_i(t)$ is vector.
From Eq.(13), there exists positive constants λ_i and l_i such that the following Eq. (14) is established.

$$\left\|\boldsymbol{\delta_i}\left(t-\Delta_i\right)\right\|\le l_i\left\|\boldsymbol{\delta_i}\left(t_0-\Delta_i\right)\right\|e^{-\lambda_i(t-t_0)} \tag{14}$$

Lemma 1.The sub-observers Eq.(8) is equivalent to Eq. (15).

$$\hat{\boldsymbol{S}}_{\boldsymbol{i}}(t)=e^{A\Delta_i}\hat{\boldsymbol{S}}_{\boldsymbol{i}}\left(t-\Delta_i\right)+\int_{t-\Delta_i}^{t}e^{\boldsymbol{A}(t-\tau)}\boldsymbol{H}U_i(\tau)d\tau \tag{15}$$

Proof. For Eq.(15), after an application of Leibniz integral rule, we obtain the following equation.

$$\begin{aligned}\dot{\hat{\boldsymbol{S}}}_{\boldsymbol{i}}(t)=&e^{A\Delta_i}\dot{\hat{\boldsymbol{S}}}\left(t-\Delta_i\right)+\boldsymbol{A}\left[\hat{\boldsymbol{S}}_{\boldsymbol{i}}(t)-e^{A\Delta_i}\hat{\boldsymbol{S}}_{\boldsymbol{i}}\left(t-\Delta_i\right)\right]\\ &+\boldsymbol{H}U_i(t)-e^{A\Delta_i}\boldsymbol{H}U_i\left(t-\Delta_i\right)\\ =&\boldsymbol{AS_i}(t)+\boldsymbol{H}U_i(t)\\ &+e^{A\Delta_i}\left[\dot{\boldsymbol{S}}_{\boldsymbol{i}}\left(t-\Delta_i\right)-\boldsymbol{AS_i}\left(t-\Delta_i\right)-\boldsymbol{H}U_i\left(t-\Delta_i\right)\right]\end{aligned}$$

From there it follows, after rearranging the terms by Eq.(9), we get the following Eq. (16).

$$\dot{\hat{\boldsymbol{S}}}_{\boldsymbol{i}}(t)=\boldsymbol{A}\hat{\boldsymbol{S}}_{\boldsymbol{i}}(t)+\boldsymbol{H}U_i(t)+e^{A\Delta_i}k_i\left[\overline{y_i}(t)-\boldsymbol{C}\hat{\boldsymbol{S}}_{\boldsymbol{i}}\left(t-\Delta_i\right)\right] \tag{16}$$

The initial value of Eq. (16) depends on Eq. (9). This concludes Lemma1.
Theorem 1. For Eq.(7) and the designed sub-observers Eq. (8) and Eq.(9), there exist positive constants λ_i and l_i such that the Eq. (17)is established.

$$\left\|\boldsymbol{\delta_i}(t)\right\|\le l_i\left\|e^{A\Delta_i}\right\|\left\|\boldsymbol{\delta_i}\left(t_0-\Delta_i\right)\right\|e^{-\lambda_i(t-t_0)} \tag{17}$$

Thus, $\lim_{t\to\infty}\boldsymbol{\delta_i}(t)=0\Rightarrow\hat{\boldsymbol{S}}_{\boldsymbol{i}}(t)\to\boldsymbol{S_i}(t)$.

Proof. For Eq.(7), the following Eq. (18)is obtained.

$$\boldsymbol{S_i}(t)=e^{A\Delta_i}\boldsymbol{S_i}\left(t-\Delta_i\right)+\int_{t-\Delta_i}^{t}e^{\boldsymbol{A}(t-\tau)}\boldsymbol{H}U_i(\tau)d\tau \tag{18}$$

Error between Eq.(15) and Eq. (18)is given as Eq. (19)

$$\boldsymbol{\delta_i}(t)=e^{A\Delta_i}\boldsymbol{\delta_i}\left(t-\Delta_i\right) \tag{19}$$

Hence, we have the following equation established from Eq. (14) to Eq.(19).

$$\left\|\boldsymbol{\delta_i}(t)\right\|\le\left\|e^{A\Delta_i}\right\|\left\|\boldsymbol{\delta_i}\left(t_0-\Delta_i\right)\right\|e^{-\lambda_i(t-t_0)}$$

This concludes the proof.

2.3 Controller Design

Estimate error is given as Eq.(20).

$$\delta_{i1}\left(t-\Delta_i\right)=\hat{S}_{i1}\left(t-\Delta_i\right)-S_{i1}\left(t-\Delta_i\right) \tag{20}$$

We rewrite Eq. (22) as Eq. (21).

$$\delta_{i1}\left(t-\Delta_1\right)=\hat{S}_{i1}\left(t-\Delta_i\right)-S_{i1}\left(t-\Delta_i\right) \tag{21}$$

As that constructions of six subsystems($x, y, z, \phi, \theta, \varphi$) are equal, so by utilizing x subsystem as an example to introduce the design of controller. Estimate error is given as (22)

$$\delta_{11}\left(t-\Delta_1\right)=\hat{x}_1\left(t-\Delta_1\right)-x_1\left(t-\Delta_1\right) \tag{22}$$

Thus, $\delta_{11}\left(t-\Delta_1\right)=\hat{x}_1\left(t-\Delta_1\right)-x_1(t-\Delta_1)$

Therefore, the designed sub-observers Eq. (8) and Eq.(9) can be equivalent to Eq.(23).

$$\begin{cases}\dot{\hat{x}}_1\left(t\right)=\hat{x}_2\left(t\right)-\left(k_{11}-\Delta_1 k_{12}\right)\delta_{11}\left(t-\Delta_1\right)\\ \dot{\hat{x}}_2\left(t\right)=U_1\left(t\right)-k_{12}\delta_{11}\left(t-\Delta_1\right)\end{cases} \tag{23}$$

Define system trace error as Eq. (24)

$$\begin{cases}e_{11}=\hat{x}_1\left(t\right)-x_d\\ e_{12}=\hat{x}_2\left(t\right)-x_d\end{cases} \tag{24}$$

Where x_d is desired position.

Substituting Eq. (24) into Eq. (23)

$$\begin{cases}e_{11}=e_{12}-\left(k_{11}-\Delta_1 k_{12}\right)\delta_{11}\left(t-\Delta_1\right)\\ e_{12}=U_1-k_{12}\delta_{11}\left(t-\Delta_1\right)-x_d\end{cases} \tag{25}$$

By choosing assistant variable (28)

$$\eta_1=e_{12}-\gamma_1 \tag{26}$$

Note the Eq. (27)

$$\gamma_1=-k_{p1}e_{11}+\left(k_{11}-\Delta_1 k_{12}\right)\delta_{11}\left(t-\Delta_1\right) \tag{27}$$

Where $k_p>0$, substituting Eq. (27) into Eq.(25), Eq. (28) is given.

$$\begin{cases}e_{11}=\eta_1-k_{p1}e_{11}\\ \eta=U_1-k_{12}\delta_{11}\left(t-\Delta_1\right)-x_d-\gamma_1\end{cases} \tag{28}$$

Design of controller is given as Eq. (29).

$$U_1=-e_{11}-k_{d1}\eta_1+k_{12}\delta_{11}\left(t-\Delta_1\right)+x_d+\gamma_1 \tag{29}$$

Where, $k_d>0$ next, going with stability analysis. Choose the Lyapunov function

$$V=\frac{1}{2}e_{11}^2+\frac{1}{2}\eta_1^2$$

And derivative of the Lyapunov function is presented as.

$$\begin{aligned}V&=e_{11}e_{11}+\eta_1\eta_1\\ &=e_{11}\left(\eta_1-k_{p1}e_{11}\right)+\eta_1\left(U_1-k_{12}\delta_{11}\left(t-\Delta_1\right)\right)-x_d-\gamma_1\\ &=e_{11}\eta_1-k_{p1}e_{11}^2+\eta_1\left(-e_{11}-k_{d1}\eta_1+k_{12}\delta_{11}\left(t-\Delta_1\right)-k_{12}\delta_{11}\left(t-\Delta_1\right)\right)\\ &=e_{11}\eta_1-k_{p1}e_{11}^2-e_{11}\eta_1-k_{d1}\eta_1^2\\ &=\left(-k_{p1}e_{11}^2-k_{d1}\eta_1^2\right)\le 0\end{aligned}$$

Theorem 2: *Lyapunov Stability*

For continuous nonlinear time-invariant autonomous system. Only when $e_{11}=\eta=0$ thus, $\dot{V}=0$ .So, the system is stable. Similarly, the other five controllers presented in Eq. (30) are obtained.

$$\begin{aligned}&\begin{cases}U_2=-e_{21}-k_{d2}\eta_2+k_{22}\delta_{21}\left(t-\Delta_2\right)+y_d+\gamma_2\\ \eta_2=e_{22}-\gamma_2\\ \gamma_2=-k_{p2}e_{21}+\left(k_{21}-\Delta_2 k_{22}\right)\delta_{21}\left(t-\Delta_2\right)\end{cases}\\ &\begin{cases}U_3=-e_{31}-k_{d3}\eta_3+k_{32}\delta_{31}\left(t-\Delta_3\right)+z_d+\gamma_3\\ \eta_3=e_{32}-\gamma_3\\ \gamma_3=-k_{p3}e_{31}+\left(k_{31}-\Delta_3 k_{32}\right)\delta_{31}\left(t-\Delta_3\right)\end{cases}\\ &\begin{cases}U_4=-e_{41}-k_{d4}\eta_4+k_{42}\delta_{41}\left(t-\Delta_4\right)+\phi_d+\gamma_4\\ \eta_4=e_{42}-\gamma_4\\ \gamma_4=-k_{p4}e_{41}+\left(k_{41}-\Delta_4 k_{42}\right)\delta_{41}\left(t-\Delta_4\right)\end{cases}\\ &\begin{cases}U_5=-e_{51}-k_{d5}\eta_5+k_{52}\delta_{51}\left(t-\Delta_5\right)+\theta_d+\gamma_5\\ \eta_5=e_{52}-\gamma_5\\ \gamma_5=-k_{p5}e_{51}+\left(k_{51}-\Delta_5 k_{52}\right)\delta_{51}\left(t-\Delta_5\right)\end{cases}\\ &\begin{cases}U_6=-e_{61}-k_{d6}\eta_6+k_{62}\delta_{61}\left(t-\Delta_6\right)+\varphi_d+\gamma_6\\ \eta_6=e_{62}-\gamma_6\\ \gamma_6=-k_{p6}e_{61}+\left(k_{61}-\Delta_6 k_{62}\right)\delta_{61}\left(t-\Delta_6\right)\end{cases}\end{aligned} \tag{30}$$

3 Simulation Results

In order to verify the effectiveness of the proposed method in the presence of system parameter uncertainties, we evaluated our solution in lots of simulations. The corresponding system and controller parameters are tabulated in Table 1 and Table 2.

Table1. System parameters

Variable	Value	Units
m	2	kg
$J_x=J_y$	0.2	$kg\cdot m^2$
J_z	0.05	$kg\cdot m^2$
l	0.215	m
g	9.8	m/s^2

Table2. Control parameters

variable	value
k_{11}	2
k_{12}	1
k_{21}	2
k_{22}	1
k_{31}	2.5
k_{32}	1
k_{41}	3
k_{42}	2
k_{51}	3
k_{52}	2
k_{61}	3
k_{62}	2

The desired trajectory of x, y and z axis all are constants marked by 1.The actual trajectory can track the desired trajectory in short time of twenty seconds and tracking errors are limited in small range presented in Fig.2 and Fig.3. The control force outputs of quadrotor are plotted in Fig.4, the upper bound is 23N in the initial time in the context of the initial errors and the stable value is limited in the specified range. Fig.5 presents interspace trajectory tracking and the blue circle presented in the figure is the desired position, so we can see that the errors are limited in the allowable range.

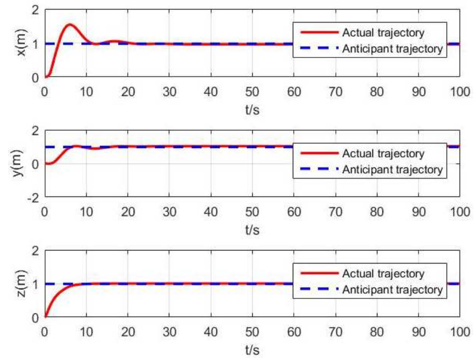

Fig.2.Tracking x, y and z curve

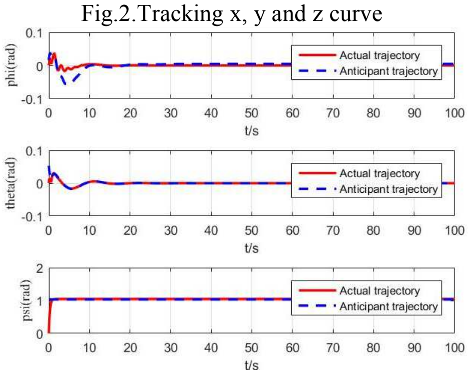

Fig.3.Attitude tracking curve

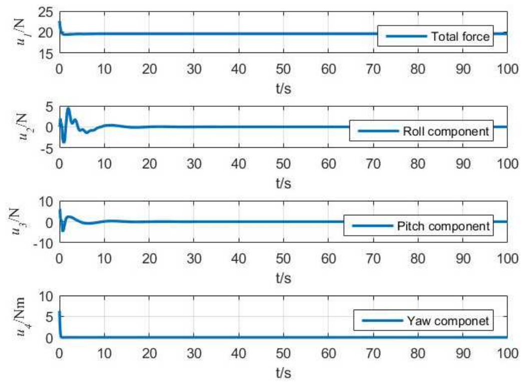

Fig.4.The output curve of control force

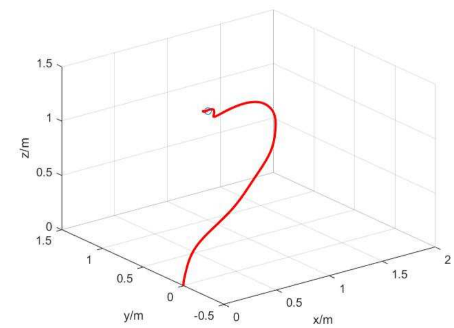

Fig.5.Trajectory of the quadrotor in the space

4 Conclusion

In the paper, we carry out a research of designing a dynamically feasible trajectory tracking strategy which drives an underactuated quadrotor to track a given reference trajectory in the aerial space. Based on the delayed-output observer, six controllers are designed to stabilize minimum phase and nonlinear phase parts, separately. Furthermore, we are pursuing the algorithm which takes external disturbance such as a gust of wind into account. We future work is aimed at planning a more dynamically feasible trajectory tracking solution that meets the need of more controllers than the number shown here. There are four controllers in the paper and the model operates in the ideal condition, so it has several disadvantages in the realistic conditions. Finally, we are interested in thinking about more novel solutions for optimizing and adapting the trajectory parameters to solve differences between the analytic model and actual.

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